

Picture this: Eric Bergoust skis down a hill on Earth. Next, he skis down an identical hill on the moon—same slope, same starting point. In both cases, gravity pulls him down the hill. The question: **What is his speed at the bottom, Earth vs. moon?**

Lunar gravity is 1/6 as strong as Earth gravity. You might naturally suppose that Bergoust's velocity on the moon would be lower by that same ratio, 1/6 or 17%, but that is wrong. In fact, his moon velocity will be $(1/6)^{1/2}$ or 41% of his Earth velocity. Here's why:

At the beginning of his run, perched at the top of the hill, Bergoust's potential energy is mgh ,

where m = the mass of the skier;
 g = the acceleration due to gravity; and
 h = the height of the hill.

He skis to the bottom, converting potential energy (mgh) to kinetic energy ($\frac{1}{2}mv^2$).

What is his velocity v ? To find out, set potential energy equal to kinetic energy and solve for v :

$$mgh = \frac{1}{2}mv^2$$

$$v = (2gh)^{1/2}$$

Now we can compare his velocity on Earth (v_E) to his velocity on the moon (v_m):

$$v_E = (2g_E h)^{1/2} \quad \text{equation (1)}$$

$$v_m = (2g_m h)^{1/2} \quad \text{equation (2)}$$

Combining equations 1 and 2 we find

$$v_m/v_E = (g_m/g_E)^{1/2} = (1/6)^{1/2} = 0.41 \quad \text{equation (3)}$$

So, the velocity at the bottom of the moon-hill is 41% the velocity at the bottom on the Earth-hill. 70 km/hr on Earth becomes 28 km/hr on the moon.

Next, we can calculate hang-time, Earth vs. moon.

According to Newton, the hang-time, t , for any ballistic trajectory is given by equation (4)

$$t = 2v\sin(\theta)/g \quad \text{equation (4)}$$

Let's compare hang time on Earth (t_E) vs. hang time on the moon (t_m).

$$t_E = 2v_E \sin(\theta) / g_E \quad \text{equation (5)}$$

$$t_m = 2v_m \sin(\theta) / g_m \quad \text{equation (6)}$$

Combining equations 3, 5 and 6, we find

$$t_m / t_E = (v_m / v_E) (g_E / g_m) = 6^{1/2} = 2.4$$

Hang time on the moon is 2.4 times that on Earth. 3 seconds on Earth becomes 7.2 seconds on the moon.

Caveat emptor: The analysis, above, contains several simplifying assumptions: (1) We assume that snow and moondust are equally frictionless surfaces, which is wrong, especially for moondust, which is expected to be much more abrasive than snow. The math is sound, however, if Bergoust uses super-slick frictionless skis for his runs on the moon. (2) We ignore any push Bergoust gives himself at the top of the hill. His velocity, the analysis asserts, is due entirely to the pull of gravity. And (3) we ignore the effects of air resistance, which is fine on the airless moon, but imprecise on Earth. Are these oversimplifications? Future lunar Olympics will tell.

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